JOURNAL OF PROPULSION AND POWER
Vol. 14, No. 4, July–August 1998

Simplified Approach to Performance Evaluation of Nuclear and Electrical Propulsion Systems

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A new simplified approach is developed to analyze the performance of alternative propulsion systems with an additional power source, such as augmented catalytic, nuclear, and electrical propulsion systems. The optimum performance characteristics are calculated for missions with a fixed time of flight, constant thrust, or constant mass flow rate, considering the effects of propellant density, tankage fraction, and chemical enthalpy of propellants. A sensitivity analysis is employed to determine the influence of different parameters on mass efficiency.

Nomenclature

C_p = specific heat at constant pressure
C_s = structural coefficient
c_0, c = constants
F = thrust
f_{tk} = tankage fraction
h = enthalpy
Isp = specific impulse
K = constant
K_1, K_2 = constants
M = molecular weight
$\dot{m}$ = mass flow rate
m_l = payload mass
m_{prop} = initial propellant mass
m_{ps} = power source mass
m_{tk} = tankage mass
m_0 = initial mass of a spacecraft
P_{tk} = tank internal pressure
R_0 = universal gas constant
r = sensitivity ratio
T_c = chamber temperature
T_0 = stagnation temperature
t_f = time of flight
V_e = maximum ejection velocity of the propellant gases
x = nondimensional augmented specific impulse
$\bar{x}$ = nondimensional specific impulse
W = power input to propellant
W_p = maximum available power
α = specific mass of power source
γ = ratio of specific heats
ΔV = characteristic velocity increment
ε = sensitivity coefficient
η = mass efficiency
η_P = efficiency of electrical–thermal power conversion
ρ_{prop} = propellant density
ρ_{tk} = density of tank material
σ_{tk} = yielding strength for the tank material
ϕ = O/F mass ratio

Subscripts

a = condition when heat is added
e = condition at nozzle exit
f = fuel
o = oxidizer, initial, dummy
p = power
prop = propellant
ps = power system
tk = tank
1 = condition at chamber entrance
2 = condition at chamber exit

Introduction

MISSIONS with very large changes in vehicle velocity require the use of very energetic propellants. But even with these propellants only a small payload fraction can be obtained, because the propellant mass fraction can be very large. The classical expression derived by Tsiolkovski for the rocket mass ratio indicates that the exhaust velocities have to be comparable or larger than the characteristic velocities to allow a significant payload ratio.

Purely chemical rockets are limited to specific impulses of about 450 s, for liquid hydrogen and liquid oxygen. If higher specific impulses (proportional to the exhaust velocities) are to be obtained, an extra energy source other than the propellant chemical bonds must be utilized.

Alternative schemes for propulsion are investigated by means of the addition of energy to the propellant gases before exhaustion, to produce higher specific impulses. The energy could be supplied by thermal heating of propellant gases, from a nuclear reactor, solar panels, solar concentrators, radiative sources, laser power, or any other of a variety of schemes.

The propellant selection can be made by use of the approximate expression

$$V_e \cong \left(\frac{2\gamma}{\gamma - 1}\frac{R_0}{M}T_c\right)^{1/2} \tag{1}$$

This expression indicates the use of propellants with low molecular weight, such as hydrogen, which yields the highest exhaust velocities. Material limitations, however, do not allow operation with chamber temperatures above 2500 K for extended periods of time. Consequently, the maximum attainable specific impulse, $Isp \approx V_e/g_0$, would be approximately 870 s, for H_2 with $\gamma = 1.4$ and $g_0 = 9.81$ m/s^2. If γ is taken as 1.3, corresponding to 2500 K, the maximum attainable Isp would be 966 s, which is 11% higher. Use of a cooling system would allow operation at higher bulk temperatures, implying higher specific impulses. Therefore, in the case of electrothermal thrusters, augmented catalytic thrusters and nuclear rockets, and other electrical propulsion systems, the difference between augmented and nonaugmented specific impulses can be relatively small.[1] The chemical enthalpy of propellants can be a

Received July 11, 1997; revision received Jan. 5, 1998; accepted for publication Jan. 15, 1998.

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significant part of the exhaust kinetic energy, different from what is generally assumed for electrical propulsion analysis.

Several parameters and approaches can be utilized to evaluate a mission.[2–4] The present analysis considers the performance of propulsion systems with an additional power source. The mass of the power source can decrease the payload to be transported, despite the augmented ejection velocities. Therefore, the mass efficiency, or payload ratio, is the most appropriate parameter to compare different configurations of augmented propulsion systems.[2,3,5,6] A comparison is made of the space vehicle mass efficiency vs the additional power input, for a given time of flight, thrust level, or fuel mass consumption rate, considering the propellant chemical enthalpy and the propellant density.

Theoretical Analysis

The mass efficiency is defined as the ratio between the payload mass and the total initial mass, for a given mission with a characteristic velocity increment ΔV. The relation between mass efficiency and ΔV is obtained from an analysis of the mass distribution in the space vehicle. The mass distribution is given by

$$m_0 = m_l + m_{ps} + m_{prop} + m_{tk} \tag{2}$$

The payload includes all of the structure mass not directly related to tankage. When part or all of the additional energy to the propellant comes from a source that belongs to the payload, as in the case of a solar panel, the corresponding power source mass m_{ps} can be decreased or eliminated.

For a single propellant the sum of the propellant mass and the tankage mass for a spherical tank with thin walls is given by

$$m_{prop} + m_{tk} = m_{prop}\left[1 + (1 + C_s)\frac{3P_{tk}\rho_{tk}}{2\sigma_{tk}\rho_{prop}}\right] \tag{3}$$

The structural coefficient C_s can be modified for cylindrical tanks, and includes the support and accessories for the tank, with values between 0.1 and 0.3.

A tankage fraction f_{tk} for a single propellant is defined as

$$f_{tk} = (1 + C_s)\frac{3P_{tk}\rho_{tk}}{2\sigma_{tk}\rho_{prop}} \tag{4}$$

For bipropellant systems the total tankage fraction is obtained from

$$f_{tk} = \frac{f_{tk,f} + \phi f_{tk,o}}{1 + \phi} \tag{5}$$

The power source mass is proportional to the maximum available power,[2,6] i.e.,

$$m_{ps} = \alpha W_p \tag{6}$$

Any other type of relation between m_{ps} and W_p can be considered, leading to more complex results. For solar panels,[7] α = 15 kg/kW, and for the NERVA nuclear engine,[1] α = 7.5 kg/kW. The power input to the propellant is given by $W = \eta_p W_p$, where η_p is the conversion efficiency of electrical to thermal power in the case of solar panels. For nuclear propulsion or other types, such as radiative source propulsion, η_p is unity. From now on W will be used instead of W_p. It is assumed that solar propulsion only occurs during sunlit periods in the trajectory. Alternatively, α could be defined to include an energy storage term.

The simplified analysis is not detailed to the point of including other parameters such as electrode voltage, current, magnetic field strength, energy loss per ion (eV), etc. However, any relationship linking the electrical power characteristics and the power system mass can be considered.

Inserting Eqs. (3) and (4) into Eq. (2) and dividing by m_0, it follows that

$$\frac{m_l}{m_0} = 1 - \alpha\frac{W}{m_0} - (1 + f_{tk})\frac{m_{prop}}{m_0} \tag{7}$$

The ratio m_{prop}/m_0 is derived from the momentum equation applied to the space vehicle, which results in the classical relation

$$\Delta V = g_0 Isp\, \ell n\frac{m_0}{m_f} \tag{8}$$

where $m_f = m_0 - m_{prop}$ is the final mass of the space vehicle after the propellant is consumed, and Isp is defined by $Isp = F/\dot{m}g_0$ ($\dot{m}$ is the mass flux of propellants). Therefore, the ratio m_{prop}/m_0 becomes

$$\frac{m_{prop}}{m_0} = 1 - \exp\left(-\frac{\Delta V}{g_0 Isp}\right) \tag{9}$$

which when substituted into Eq. (7), yields

$$\eta = 1 - \alpha\frac{W}{m_0} - (1 + f_{tk})\left[1 - \exp\left(-\frac{\Delta V}{g_0 Isp}\right)\right] \tag{10}$$

where $\eta = m_l/m_0$ is the mass efficiency or payload ratio.

Augmented Specific Impulse

By applying the first law of thermodynamics for the flow in the thruster, the augmented exhaust velocities can be related to the power input by the expression

$$V_{e,a}^2 = \left(V_{e,opt}^2 + \frac{2W}{\dot{m}_a}\right)\left[1 - \left(\frac{P_e}{P_{01}}\right)^{(\gamma-1)/\gamma}\right] \tag{11}$$

where the subscript a designates conditions when heating is supplied, V_e is the exhaust velocity, P_e is the pressure at the nozzle exit, and P_{01} is the chamber pressure. The pressure ratio P_e/P_{01} depends only on the area ratio of the nozzle and it is evaluated with no heating. Equation (11) was derived assuming a one-dimensional flow of a perfect gas with a constant specific heat ratio, given by γ. A schematic view of the heating process is shown in Fig. 1.

The propellant gas enters the chamber with a stagnation temperature $T_{01,a}$, leaves it with a stagnation temperature $T_{02,a}$, and its internal energy is converted into kinetic energy by the nozzle, with an exhaust velocity $V_{e,a}$.

In the case of optimum expansion in vacuum, the expression [Eq. (11)] for the augmented system is simplified to

$$V_{e,a}^2 = V_e^2 + 2W/\dot{m}_a \tag{12}$$

where the subscript *opt* has been dropped. If $2W/\dot{m}_a >> V_e^2$, it means that the propellant chemical energy is not significant compared to the additional power input, because V_e depends

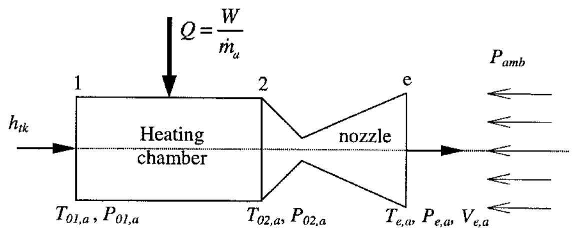

Fig. 1 Scheme of the thruster flow.

only on the chemical energy of propellants for optimum expansion.

Considering that $\dot{m}_a = m_{prop}/t_f$, it follows that

$$\dot{m}_a = (m_0/t_f)[1 - \exp(-1/x)] \quad (13)$$

where $x = g_0 Isp_a/\Delta V$. Therefore, Eq. (12) can be rewritten as

$$V_{e,a}^2 = V_e^2 + (2t_f W/m_0)[1 - \exp(-1/x)]^{-1} \quad (14)$$

and, consequently, the augmented specific impulse is given by

$$Isp_a = \{Isp^2 + (2t_f W/g_0^2 m_0)[1 - \exp(-1/x)]^{-1}\}^{1/2} \quad (15)$$

yielding the following expression for the specific power consumption:

$$W/m_0 = (g_0^2/2t_f)(Isp_a^2 - Isp^2)[1 - \exp(-1/x)] \quad (16)$$

Optimum Performance

Substituting Eq. (16) into Eq. (10) and defining the variables $\bar{x} = g_0 Isp/\Delta V$ and $K = 2t_f(1 + f_{tk})/(\alpha \Delta V^2)$, gives the following expression for the mass efficiency:

$$\eta = 1 - (1 + f_{tk})[1 + (x^2 - \bar{x}^2)/K][1 - \exp(-1/x)] \quad (17)$$

and Eq. (16) can also be rewritten as

$$\omega = [(1 + f_{tk})/\alpha K](x^2 - \bar{x}^2)[1 - \exp(-1/x)] \quad (18)$$

where $\omega = W/m_0$. The maximum mass efficiency for a given augmented specific impulse can be found by taking the derivative of η with respect to x and equaling to zero, yielding the relation

$$2x^3(e^{1/x} - 1) = K + x^2 - \bar{x}^2 \quad (19)$$

and, once the chemical enthalpy of the propellants is neglected, it simplifies to $2x^3(e^{1/x} - 1) = K + x^2$, yielding lower values for the optimum augmented specific impulse and higher values of optimum mass efficiency, if Eq. (19) has a root. It can be verified that Eq. (19) can have zero, one, or two solutions. For the case with two solutions only the larger root presents a negative second derivative of η and there exists a maximum efficiency.

Figure 2 presents the function $f(x) = 2x^3(e^{1/x} - 1) - x^2$, obtained from Eq. (19) with $f(x) = K - \bar{x}^2$, and also the function $g(x) = (3 + 1/x)e^{-1/x} - (3 - 1/x)$, which is related to the second derivative of the mass efficiency, i.e., $d^2\eta/dx^2 = 2(1 + f_{tk})g(x)/K$. It is observed that for $K - \bar{x}^2 > 1.27$, approximately, there will exist a maximum efficiency.

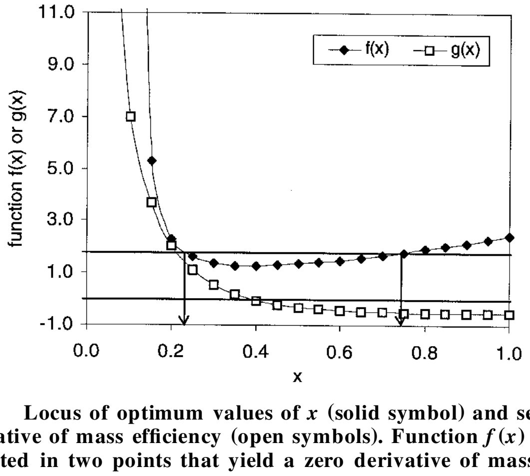

Fig. 2 Locus of optimum values of x (solid symbol) and second derivative of mass efficiency (open symbols). Function $f(x)$ is intersected in two points that yield a zero derivative of mass efficiency.

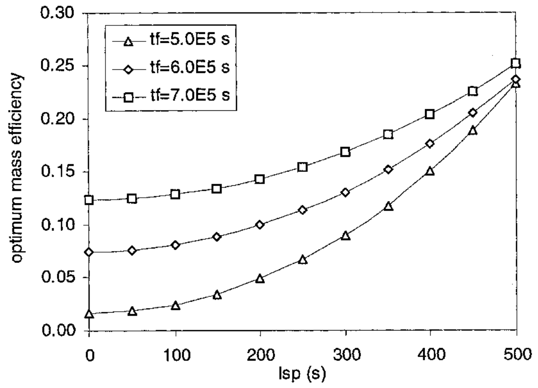

Fig. 3 Optimum mass efficiency vs *Isp* for fixed flight time.

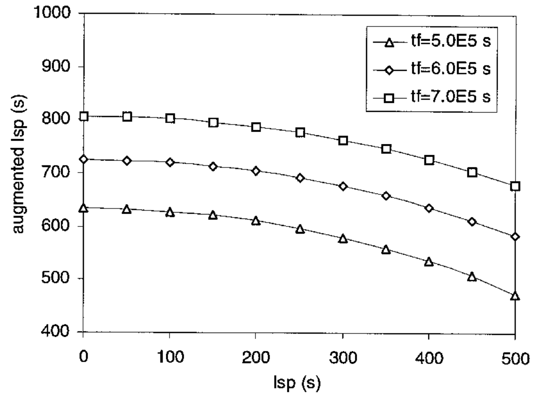

Fig. 4 Optimum augmented specific impulse vs *Isp* for fixed flight time.

Figure 3 shows the optimum values of mass efficiency for a mission with $\Delta V = 5000$ m/s, $\alpha = 15$ kg/kW, and $f_{tk} = 0.2$. Figure 4 shows the corresponding values of specific impulses for the same data.

It can be seen in Fig. 3 that increasing values of nonaugmented specific impulses can lead to significantly higher optimum mass efficiencies and that increasing flight times lead to significantly higher mass efficiencies. It is shown in Fig. 4 that higher nonaugmented specific impulses lead to lower optimum augmented specific impulses and, consequently, a lower power input is required, as seen from Eq. (16).

The previous analysis assumed a given time of flight and, for a given power input, the mass efficiency can be determined from Eq. (17). An alternative approach is to consider an augmented propulsion system with a fixed mass flow rate or a fixed thrust level. In these cases different expressions for the mass efficiency and power consumption are obtained.[5] The optimum conditions with these assumptions are presented next.

Constant Thrust and Constant Mass Flow Rate Performance

The ratio of specific impulses can also be obtained from the expression

$$\frac{Isp_a}{Isp} \cong \frac{V_{e,a}}{V_e} = \left(1 + \frac{W}{\dot{m} C_p T_{01}}\right)^{1/2} \quad (20)$$

Assuming a constant mass flow rate for both augmented and nonaugmented thrusters, with $\dot{m}_a = \dot{m}$, e.g., by changing the injection parameters, and substituting the relations $\dot{m} = F/(g_0 Isp)$ and $Isp^2 = 2C_p T_{01}/g_0^2$ into Eq. (20), gives

$$Isp_a = \{Isp[(2W/Fg_0) + Isp]\}^{1/2} \quad (21)$$

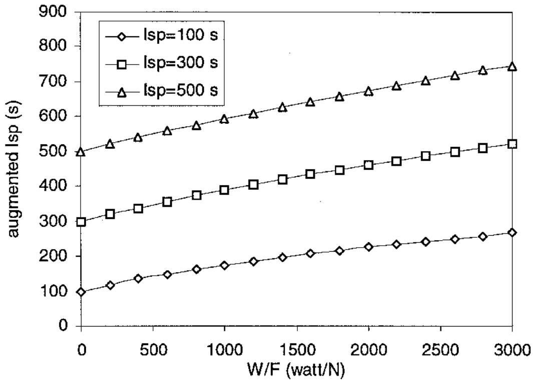

Fig. 5 Augmented specific impulses vs power/thrust ratio, for constant mass flow rate.

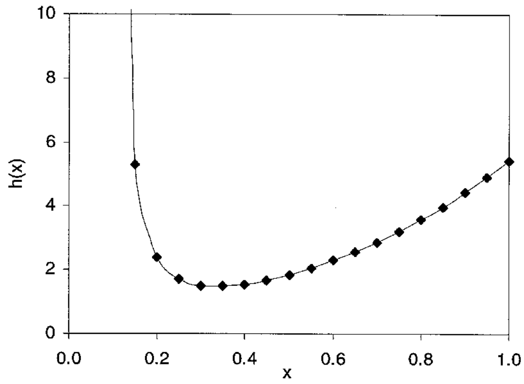

Fig. 6 Locus of optimum values of *x* for a constant mass flow engine. Optimum values of *x* yield optimum efficiencies.

or, alternatively, assuming a constant thrust level, $F_a = F$, also by changing appropriately the injection parameters and considering $Isp^2 = 2C_pT_{01}/g_0^2$, it follows that

$$Isp_a = (W/Fg_0) + [(W/Fg_0)^2 + Isp^2]^{1/2} \tag{22}$$

Figure 5 displays augmented specific impulses vs power/thrust ratio for different propellants or different nonaugmented specific impulses, for constant mass flow rate.

The power input for the constant thrust case is obtained from

$$W = (F\Delta V/2m_0)(x^2 - \bar{x}^2)/x \tag{23}$$

and, for thrusters keeping a constant mass flow rate, the power input is given by

$$W = (F\Delta V/2m_0)(x^2 - \bar{x}^2)/\bar{x} \tag{24}$$

The flight times for constant thrust and constant mass flow rate will be modified and they will depend on the power input and specific impulses obtained. For a constant thrust engine the flight time will be given by $t_f = (m_0\Delta V/F)x[1 - \exp(-1/x)]$, whereas for constant mass flow rate the flight time will be $t_f = (m_0\Delta V/F)\bar{x}[1 - \exp(-1/x)]$.

In the case of constant mass flow rate, the augmented thrust will be given by $F_a = FIsp_a/Isp$, and for constant thrust the mass flow rate will be given by the relation $\dot{m}_a = \dot{m}Isp/Isp_a$.

The mass efficiency for a specific mission can be obtained for each case by substituting Eqs. (23) and (24) into Eq. (10), yielding

$$\eta = 1 - K_1(x^2 - \bar{x}^2)/x - (1 + f_{tk})[1 - \exp(-1/x)] \tag{25}$$

$$\eta = 1 - K_1(x^2 - \bar{x}^2)/\bar{x} - (1 + f_{tk})[1 - \exp(-1/x)] \tag{26}$$

for constant thrust and constant mass flow rate, respectively, with $K_1 = \alpha F\Delta V/(2m_0)$.

The optimum mass efficiencies are obtained once more by taking derivatives with respect to x and equaling to zero, yielding the relation $(x^2 + \bar{x}^2)e^{1/x} = K_2$ for constant thrust, and $2x^3e^{1/x} = K_2\bar{x}$, for constant mass flow rate, with $K_2 = (1 + f_{tk})/K_1$.

Figure 6 displays the function $h(x) = 2x^3e^{1/x} = K_2\bar{x}$, for a constant mass flow rate engine. It indicates that for $K_2\bar{x} > 1.49$, approximately, there will exist a local minimum and a local maximum of mass efficiency. A similar behavior should occur for a constant thrust engine.

Figure 7 shows values of optimum mass efficiency for the constant thrust case, with $\Delta V = 18{,}000$ m/s, $m_0 = 500$ kg, $\alpha = 15$ kg/kW, and $f_{tk} = 0.2$. It can be easily verified that neglecting

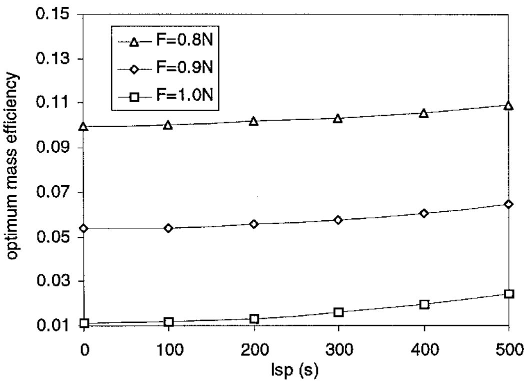

Fig. 7 Optimum mass efficiencies vs *Isp* for constant thrust.

the propellant chemical energy leads to lower optimum mass efficiencies and higher optimum values for the augmented specific impulses.

It can be seen in Fig. 7 that there is an increase in mass efficiency when more energetic propellants are employed. For the 1.0 N thruster, the mass efficiency is three times larger when compared to the case with negligible chemical enthalpy. It is also verified in Fig. 7 that increasing nonaugmented specific impulses, or increasing chemical enthalpies, lead to decreasing optimum augmented specific impulses, with a lower power consumption.

Sensitivity Analysis

The use of different O/F ratios or special propellants to obtain higher specific impulses will cause, in general, a change in propellant density and tank pressure. In the previous examples the f_{tk} was assumed constant, and then relatively simple expressions for optimum performance could be obtained. For a given standard configuration, the effects of changes in propellant characteristics and other parameters can be evaluated by means of a sensitivity analysis of the propulsion system. The sensitivity to any parameter is obtained by differentiating the expression for mass efficiency, e.g.,

$$\frac{d\eta}{\eta} = \varepsilon_{\rho_{prop}}\frac{d\rho_{prop}}{\rho_{prop}} + \varepsilon_{Isp}\frac{dIsp}{Isp} + \varepsilon_{P_{tk}}\frac{dP_{tk}}{P_{tk}} \tag{27}$$

where ε_i ($i = \rho_{prop}$, Isp, or P_{tk}) are the sensitivity coefficients to the propellant density, nonaugmented specific impulse (chemical enthalpy), and tank pressure, respectively, given by

$$\varepsilon_\rho = \frac{\rho}{\eta}\frac{\partial\eta}{\partial\rho}, \qquad \varepsilon_{Isp_a} = \frac{Isp}{\eta}\frac{\partial\eta}{\partial Isp}, \qquad \varepsilon_{P_{tk}} = \frac{P_{tk}}{\eta}\frac{\partial\eta}{\partial P_{tk}} \tag{28}$$

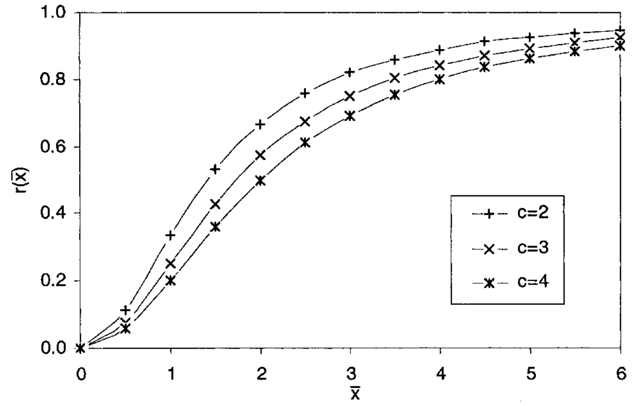

Fig. 8 Sensitivity coefficient ratio $r = \varepsilon_{Isp}/\varepsilon_{\rho_{prop}}$, with $r(\bar{x} \to \infty) = 1$.

To obtain expressions for the sensitivity coefficients in the case of fixed flight time it is necessary to differentiate Eq. (17) for mass efficiency, and Eq. (18) for power input, yielding

$$\varepsilon_\rho = (f_{tk}/\eta)[1 - \exp(-1/x)] \tag{29}$$

$$\varepsilon_{Isp} = \frac{1 + f_{tk}}{\eta} \frac{\bar{x}^2[1 - \exp(-1/x)]}{x^3[\exp(1/x) - 1] - x^2 + \bar{x}^2} \tag{30}$$

$$\varepsilon_{P_{tk}} = -(f_{tk}/\eta)[1 - \exp(-1/x)] \tag{31}$$

Equations (29–31) allow the evaluation of the effects on mass efficiency of differential changes in propellant density, nonaugmented specific impulses, and tank pressure. It can be observed that tank pressure and propellant density have equal influence but are of opposite sign: higher tank pressure always decreases mass efficiency and higher propellant density always increases mass efficiency, independent of any other condition. Contrarily, nonaugmented specific impulses will affect mass efficiency depending on x values. Another approach for analyzing the relation between specific impulse and propellant density is to define the sensitivity ratio $r = \varepsilon_{Isp}/\varepsilon_{\rho_{prop}}$, given by

$$r = \frac{1 + f_{tk}}{f_{tk}} \frac{\bar{x}^2}{x^2[x \exp(1/x) - x - 1] + \bar{x}^2} \tag{32}$$

with the general form $r(\bar{x}) = c_0\bar{x}^2/(c + \bar{x}^2)$, where

$$c_0 = (1 + f_{tk})/f_{tk}, \qquad c = x^2[x \exp(1/x) - x - 1] \tag{33}$$

are real positive constants for a given propulsion system configuration.

Figure 8 shows the curve $r(\bar{x})$, for $c_0 = 1$ and $c = 2$, 3, and 4. It can be seen in Fig. 8 that $r(\bar{x} \to \infty) = c_0 = 1$ and $r(0) = 0$. The propulsion system becomes more sensitive for increasing values of c_0, which is equivalent to an increasing propellant density, and more sensitive for decreasing values of c, which is equivalent to an increasing augmented specific impulse. For low values of the chemical enthalpy the system has a small sensitivity to changes in the propellant, whereas for high values the system presents a larger sensitivity, which increases asymptotically to c_0.

The sensitivity analysis can be employed to evaluate the influence of many other parameters. For example, the effect of a small increase in power input or *O*/*F* ratio on the payload mass can be calculated by using appropriate sensitivity coefficients.

Conclusions

A comprehensive method has been developed and employed to evaluate the performance of augmented propulsion systems, with a separate power source. Expressions for the optimum mass efficiency and optimum augmented specific impulse have been determined for different mission specifications. The effects of chemical enthalpy and tankage fraction were considered for missions with a fixed time of flight, constant thrust, or constant mass flow rate. Conditions for obtaining increasing mass efficiencies for a given mission have been determined and it has been demonstrated that optimum specific impulses and power consumption can decrease significantly by using more energetic propellants for typical missions. A sensitivity analysis has shown the influence of different parameters on mass efficiency.

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